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Event-Triggered Control for Stabilization of Semilinear N-D Reaction-Convection-Diffusion PDE by Switching

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ABSTRACT

This paper addresses a switched sampled-data control design for stabilization of the N-dimensional (N-D) semilinear heat equation with a mobile actuator. It is supposed that discrete-time averaged measurements are available. The system is known to be stabilizable by the static output-feedback employing several distributed in space actuators and sensors, but is not stabilizable by only one of the actuator-sensor pairs. Does there exist a switching stabilizing static output-feedback such that at all times, only one actuator-sensor pair is active? In our recent paper we gave a positive answer and found the appropriate switching sampled-data time-triggered control law for the one-dimensional (1-D) case. In this paper, to enlarge the time between switching (which means to reduce the frequency of actuator moving to the active domain), we present an event-triggered control for stabilization by switching. Moreover, we extend the results for stabilization by switching to the N-D case. Numerical examples show that the switching-based event-triggered controller essentially decreases the frequency of the actuator moving compared to the time-triggered controller, reducing operating costs.

1 | Introduction

Switching control for partial differential equations (PDEs) has attracted extensive attention recently (see References [1–8]). In Reference [5], the following problem was formulated: "Assuming that one can control a system using two or more actuators, does there exist a control strategy such that at all times, only one actuator is active?" A positive answer for some PDEs subject to corresponding switching laws was given in Reference [5]. The implementation of optimal and switching policies of spatially scheduled actuators was suggested for PDEs (see Reference [6]). In Reference [7], intermittent control of the reaction-diffusion equation by time-dependent switching between all working pairs of collocated mobile actuators and sensors and the rest (all not working) has been studied. In Reference [8], the integrated design of switching control and mobile actuator/sensor guidance was proposed to exponentially stabilize the linear reaction-diffusion equation. Note that the methods mentioned above for switching control or for control by mobile actuators and sensors may be inefficient for the unstable open-loop systems.

Event-triggering mechanism (ETM) can be used to reduce the network workload (see References [9–19]). Its basic idea is to send the signal only when the discrepancy between the current signal value and the signal, which was last transmitted is large enough. Due to potential superior performance as well as possible advantages in the stability analysis, ETM for PDEs has attracted extensive attention. Periodic ETM for distributed control of 1-D semilinear diffusion PDEs was presented in Reference [12] effectively reducing the number of spatially distributed measurements transmitted through a communication network in the numerical example. Boundary continuous event-based control via the backstepping method was suggested for 1-D linear hyperbolic systems of conservation laws in Reference [13] and for 1-D linear heat equations in Reference [14]. Continuous ETM for distributed control of 1-D nonlinear Korteweg-de Vries (KdV) equation was introduced in Reference [15]. Continuous ETM was presented for adaptive output-feedback boundary control of a hyperbolic PDE-Ordinary Differential Equation (ODE) coupled system in Reference [16], for boundary control of highly re-entrant manufacturing systems modeled as a nonlinear hyperbolic PDE in Reference [17], for the wave equation in Reference [18], and for boundary control of semilinear parabolic PDEs with non-collocated distributed event-triggered observations in Reference [19]. In all the continuous ETMs for PDEs, one of the main challenges is to prove the avoidance of the Zeno behavior. To reduce the workload, in the present paper we introduce event-triggered control for stabilization by switching. Moreover, we extend stabilization by switching to N-D PDEs.

In our recent paper [20], time-triggered sampled-data stabilization by switching of 1-D parabolic PDEs was suggested. Although the above time-triggered switching control improves the performance with reduced operating and production costs, this method also increases the system cost since actuator switching happens at fixed times regardless of whether the switching is necessary or not. Therefore, the event-triggered switching control has been developed to reduce the unnecessary switching cost and wear, and thus improve the utilization rate of system resources (see References [9, 21, 22]). In References [21, 22], event-driven schemes were introduced to reduce the switching cost and frequency of signal transmissions for 1-D parabolic distributed parameter systems. Note that ETM with switching for the stabilization of semilinear N-D parabolic distributed parameter systems by moving actuators, is still an open problem, which motivates our study.

This work addresses an event-triggered control design for stabilization by switching of N-D reaction-convection-diffusion equation under the Dirichlet/Neumann boundary conditions with spatially scheduled actuators. Constructive conditions are derived to ensure that the resulting closed-loop system is exponentially stable by means of the Lyapunov approach. The main contributions of this paper can be summarized as follows:

- In this paper, we extend our recent result on switching-based stabilization of the 1-D diffusion-reaction equation to a more general N-D reaction-convection-diffusion equation. Extension of stabilization from 1-D to N-D PDEs under averaged measurements is challenging due to the high dimension, including the well-posedness proof and stability analysis. For example, the result in Reference [23] cannot be extended to the N-D case, which motivates our work.
- We present efficient ETM for switching control-ETM significantly reduces the number of sent measurements compared to time-triggered control in the numerical example.

A remainder of the paper is organized as follows. Section 2 introduces some useful inequalities. In Section 3, a dynamic event-triggered control law is suggested to stabilize the semilinear N-D reaction-convection-diffusion equation by switching. The proposed control law is based on the averaged measurements. Sections 4 and 5 are devoted to illustrate our main theoretical results. A numerical example is given to show the effectiveness of the proposed theoretical statements in Section 6. Finally, concluding remarks comprise Section 7.

Notation. Matrix $R > 0$ represents that R is symmetric positive definite. I denotes the identity of appropriate dimensions. $\mathcal{L}^2(\Omega)$ stands for the Hilbert space of square integrable scalar functions $f(x)$ on $\Omega \subset \mathbb{R}^N$ with the corresponding norm $\|f\|_{\mathcal{L}^2(\Omega)} = \left[\int_\Omega f^2(x)dx\right]^{\frac{1}{2}}$. $\mathcal{L}^\infty(\Omega)$ denotes the space of essentially bounded function $f(x)$ on Ω with the corresponding norm $\|f\|_{\mathcal{L}^\infty(\Omega)} = \text{esssup}_{x\in\Omega}|f(x)|$. The Sobolev space $H^k(\Omega)$ with $k \in \mathbb{Z}$ is defined as $H^k(\Omega) = \{f : D^\alpha f \in \mathcal{L}^2(\Omega),\ \forall\ 0 \le |\alpha| \le k\}$ with norm $\|f\|_{H^k(\Omega)} = \{\sum_{0\le|\alpha|\le k} \|D^\alpha f\|^2_{\mathcal{L}^2(\Omega)}\}^{\frac{1}{2}}$. For $z(x) = \left[z^1(x), \dots, z^M(x)\right]^T \in \mathbb{R}^M$, $\nabla_x z^m = \left[\frac{\partial z^m}{\partial x_1}, \dots, \frac{\partial z^m}{\partial x_N}\right]^T$ and $\nabla_x z = [\nabla_x z^1, \dots, \nabla_x z^M] \in \mathbb{R}^{NM}$.

2 | Useful Inequalities

Lemma 1. (Wirtinger's inequality [24, 25]). *For $\Omega = [0, L]^N$, let $z \in H^1(\Omega)$ and $z : \Omega \to \mathbb{R}$ with $z|_{\partial\Omega} = 0$. Then the following inequality holds*:

$$\|z\|^2_{\mathcal{L}^2(\Omega)} \le \frac{L^2}{N\pi^2}\|\nabla_x z\|^2_{\mathcal{L}^2(\Omega)}$$

Lemma 2. (Poincaré's inequality [24, 26]). *For $\Omega = [0, L]^N$, let $z \in H^1(\Omega)$ and $z : \Omega \to \mathbb{R}$ with $\int_\Omega z(x)dx = 0$. Then*

$$\frac{\pi^2}{NL^2}\|z\|^2_{\mathcal{L}^2(\Omega)} \le \|\nabla_x z\|^2_{\mathcal{L}^2(\Omega)}.$$

3 | Problem Formulation

Denote by Ω the N-dimensional domain $\Omega = [0, L]^N$ with the boundary $\partial\Omega$. Let

$$0 = t_0 < t_1 < \dots < t_k < t_{k+1} < \dots, \quad \lim_{k\to\infty} t_k = \infty$$

be a sequence of sampling instants that will be determined by dynamic ETM later.

Consider the following semilinear N-D reaction-convection-diffusion PDE:

$$\begin{cases} z_t(x,t) = \Delta z(x,t) - \beta\nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t)) \\ \qquad + b_{\sigma_k}(x)Bu_{\sigma_k}(t), x \in \Omega, t \in [t_k, t_{k+1}) \\ z(x,0) = z_0(x) \end{cases} \tag{1}$$

subject to Dirichlet

$$z|_{\partial\Omega} = 0,\ t > 0 \tag{2}$$

or Neumann boundary conditions

$$\frac{\partial z}{\partial n}|_{\partial\Omega} = 0,\ t > 0 \tag{3}$$

where $z(x,t) = [z^1(x,t),\ \dots,\ z^M(x,t)]^T \in \mathbb{R}^M$ is the state of the diffusion equation, $z_0(x)$ is the initial state, $u(t) = [u^1(t), \dots, u^m(t)]^T \in \mathbb{R}^m$ is the control input of the plant, σ_k is the switching signal, $A \in \mathbb{R}^{M\times M}$, $B \in \mathbb{R}^{M\times m}$ are constant matrices, and $\beta \in \mathbb{R}^{M\times NM}$ is the convection coefficient matrix. The diffusion term is given by

$$\Delta z(x,t) = \left[\Delta z^1(x,t)), \dots, \Delta z^M(x,t))\right]^T$$
$$\Delta z^m(x,t) = \sum_{k=1}^{N} \frac{\partial^2 z^m(x,t)}{\partial x_k^2},\ m = 1,2,\dots,M$$

We make the following assumptions:

- Controllability: The pair (A, B) is controllable.
- Nonlinearity: It is supposed that the function φ is of class C^1 and satisfies the following inequality:

$$\varphi^T(z(x,t))\varphi(z(x,t)) \le z^T(x,t)Qz(x,t) \tag{4}$$

  where $Q \in \mathbb{R}^{M\times M}$ is some positive definite matrix. The open-loop system (1) may become unstable if $\|Q\|$ in (4) is large enough.
- Spatial sampling: As in References [27–30], we divide Ω into N_s equal subdomains Ω_j $(j = 1, \dots, N_s)$ with $\cup_{j=1}^{N_s}\Omega_j = \Omega$. The shape functions $b_j(x)$ are chosen to be characteristic functions $b_j(x)$ of Ω_j as follows:

$$\begin{cases} b_j(x) = 0,\ x \notin \Omega_j, \\ b_j(x) = 1,\ \text{otherwise}, \end{cases} j = 1, \dots, N_s \tag{5}$$

- Measurements: Assume that sensors provide the averaged measurements:

$$y_j(t) = \frac{\int_{\Omega_j} z(x,t)dx}{|\Omega_j|} = \left(\frac{N_s}{L}\right)^N \int_{\Omega_j} z(x,t)dx,\ j = 1, \dots, N_s \tag{6}$$

- Moving time: The moving time $\delta \in (0, h_0)$ for actuators to the appropriate domain Ω_{σ_k} is taken into account similar to Reference [20]. Therefore, the length of sampling subintervals in time satisfies

$$0 < h_0 \le t_{k+1} - t_k$$

- Time sampling: Inspired by References [31, 32], the sampling instants $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \dots$, $\lim_{k\to\infty} t_k = \infty$ are determined by the following dynamic ETM with a waiting time $h_0 > 0$:

$$t_{k+1} = \min\left\{ t > t_k + h_0 \Big| \sum_{j=1}^{N_s}\left[|e_j(t)|^2 - \varepsilon|y_j(t)|^2\right] > \theta m(t) \right\} \tag{7}$$

  where $\varepsilon > 0$, $\theta > 0$, and the errors $e_j(t) = y_j(t) - y_j(t_k)$ are continuous for $t \in [t_k + h_0, t_{k+1})$. The dynamic variable $m(t)$ satisfies the following differential equation:

$$\dot m(t) = \begin{cases} -2\varepsilon_1 m(t), & t \in [t_k, t_k + h_0) \\ -\varepsilon_0 m(t) + \sum_{j=1}^{N_s}\left[\varepsilon|y_j(t)|^2 - |e_j(t)|^2\right], & t \in [t_k + h_0, t_{k+1}) \end{cases} \tag{8}$$

  for scalar parameters $\varepsilon_0 > 0$ and $\varepsilon_1 > 0$. Assume that the initial condition $m(0) = m_0 \ge 0$, which implies that $m(t) \ge 0$ for $t \in [0,\infty)$. This dynamic ETM can determine the sampling instants at which the measurements and the switching control law need to be updated, whereas the actuator starts moving to the resulting zone.

In order to take into account the moving time of actuators, we follow Reference [20] and consider additional switching between the open-loop system (when the actuator is moving) during the part of the sampling interval and the closed-loop switched system during the remaining part of the interval.

Inspired by the above, we aim at proposing a dynamic event-triggered controller for stabilization by switching of N-D reaction-convection-diffusion PDE (1) that can be implemented by

$$u_{\sigma_k}(t) = \begin{cases} 0, & t \in [t_k, t_k + \delta) \\ -Ky_{\sigma_k}(t_k), & t \in [t_k + \delta, t_{k+1}) \end{cases} \tag{9}$$

with some $K \in \mathbb{R}^{m\times M}$ that $A - BK$ is Hurwitz. The switching signal σ_k is calculated at time t_k, whereas it takes δ seconds for actuators and sensors to move to the domain Ω_{σ_k}.

Our main objective is to find an appropriate output-depending switching law. Define a characteristic function:

$$\chi_{[a,b]}(t) = \begin{cases} 1, & \text{if } t \in [a,b] \\ 0, & \text{otherwise} \end{cases}$$

Under the controller (9), the closed-loop system has the form

$$\begin{aligned} z_t(x,t) &= \Delta z(x,t) - \beta\nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t)) \\ &\quad - (1 - \chi_{[t_k,t_k+\delta]})b_{\sigma_k}(x)BK\left(\frac{N_s}{L}\right)^N \int_{\Omega_{\sigma_k}} z(x,t_k)dx \end{aligned} \tag{10}$$

subject to (2) or (3). If $b_{\sigma_k}(x)Bu_{\sigma_k}(t)$ in (1) is changed by $\sum_{j=1}^{N_s} b_j(x)Bu_j(t)$, then the corresponding closed-loop system is exponentially stable (see Reference [24]). The latter means that the average of systems (1) with $b_{\sigma_k}(x)Bu_{\sigma_k}(t)$ changed by $b_j(x)Bu_j(t)$ is stabilizable by the static output-feedback (9). Following Reference [20], define a min-type switching function by using the Lyapunov function $V(t) = \int_\Omega z^T(x,t)P_2 z(x,t)dx$ with $P_2 > 0$ according to

$$\sigma_k = \arg\min \dot V(t)$$

for $t \in [t_k + \delta, t_k + h_0)$ along the closed-loop system (10). Thus, we have

$$\dot{V}(t) = -\left(\frac{N_s}{L}\right)^N \int_{\Omega_j} z^T(x,t)dx(P_2BK + K^TB^TP_2)\int_{\Omega_j} z(x,t_k)dx + \int_\Omega z^T(x,t)(P_2A + A^TP_2)z(x,t)dx + 2\int_\Omega z^T(x,t)P_2[\Delta z(x,t) - \beta\nabla_x z(x,t) + \varphi(z(x,t))]dx \tag{11}$$

Then for small enough h_0, (11) leads to the following switching rule:

$$\arg\min \dot{V}(t) = \arg\min_j \left[-\int_{\Omega_j} z^T(x,t_k)dx(P_2BK + K^TB^TP_2)\int_{\Omega_j} z(x,t_k)dx\right] \tag{12}$$

For $[t_k + h_0, t_{k+1})$, we still choose the above switching rule. The switching rule (12) means that the σ_k-th mode is active if

$$\int_{\Omega_j} z^T(x,t_k)dx(P_2BK + K^TB^TP_2)\int_{\Omega_j} z(x,t_k)dx \leq \int_{\Omega_{\sigma_k}} z^T(x,t_k)dx(P_2BK + K^TB^TP_2)\int_{\Omega_{\sigma_k}} z(x,t_k)dx, \forall j = 1,\ldots,N_s$$

Remark 1. If (4) holds locally for $|z| \leq \sigma_0$ with some $\sigma_0 > 0$, then regional stabilization can be achieved following the arguments of Reference [35]. If (6) is changed by pointlike measurements as considered in Reference [23], a time-delay approach with appropriate Lyapunov functional seems to be not applicable for $N > 2$. Here a continuous-time ETM should be studied like considered in References [13–15] for PDEs, which can be a subject for future research.

4 | Well-Posedness of the Controlled System

In this section, we will analyze the well-posedness of the system (1) subject to (2) or (3) under the switching control law (9) by using the step method.

Define the system operator $\mathcal{A} : D(\mathcal{A}) \subset [\mathcal{L}^2(\Omega)]^M \to [\mathcal{L}^2(\Omega)]^M$ as follows:

$$\begin{cases} \mathcal{A}g = \Delta g - \beta\nabla_x g + Ag \\ D(\mathcal{A}) = [H^2(\Omega)]^M \cap [H_0^1(\Omega)]^M \end{cases}$$

It is well-known that $\mathcal{A}$ is a dissipative operator, and $\mathcal{A}$ generates an analytic semigroup $T(t)$. Operator $-\mathcal{A}$ is positive, implying that its square root $(-\mathcal{A})^{\frac{1}{2}}$ is also positive. Moreover,

$$D((-\mathcal{A})^{\frac{1}{2}}) = [H_0^1(\Omega)]^M = \left\{ g \in [H^1(\Omega)]^M \;\middle|\; g|_{\partial\Omega} = 0 \right\}$$

For $t \in [t_0, t_1]$ and the event-triggered switching control law (12), we assume that the σ_k-th mode is active. Consider the following equation:

$$\begin{cases} z_t(x,t) = \Delta z(x,t) - \beta\nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t)) \\ \qquad -b_{\sigma_k}(x)BK\left(\frac{N_s}{L}\right)^N \int_{\Omega_{\sigma_k}} z_0(x)dx \\ z|_{\partial\Omega} = 0 \\ z(x,0) = z_0(x) \end{cases} \tag{13}$$

Then the system (13) can be represented as an evolution equation:

$$\begin{cases} \frac{d}{dt}z(\cdot,t) = \mathcal{A}z(\cdot,t) + \mathcal{F}(z(\cdot,t)) \\ z(\cdot,0) = z_0(\cdot) \end{cases}$$

where the nonlinear term $\mathcal{F}$ is defined on function $z(\cdot,t)$ according to

$$\mathcal{F}(z(\cdot,t)) = \begin{cases} \varphi(z(\cdot,t)), & t \in [t_0, t_0+\delta] \\ \varphi(z(\cdot,t)) - b_{\sigma_k}(x)BK\left(\frac{N_s}{L}\right)^N \int_{\Omega_{\sigma_k}} z_0(x)dx, & t \in [t_0+\delta, t_1] \end{cases}$$

It should be noticed that the nonlinear term $\mathcal{F}$ is locally Lipschitz continuous.

Then by using Theorem 3.3.3 of Reference [33], we obtain that for any initial conditions $z_0 \in [H_0^1(\Omega)]^M$, there exists a unique strong solution on $[0,T] \subset [t_0, t_1]$:

$$\begin{aligned} &z \in C([0,T]; [H_0^1(\Omega)]^M) \cap \mathcal{L}^2([0,T]; D(\mathcal{A})) \\ &z_t \in \mathcal{L}^2([0,T]; [\mathcal{L}^2(\Omega)]^M) \end{aligned} \tag{14}$$

We will prove that under the conditions of Theorem 1, the solution is bounded, and thus is continuable to all intervals $[0,T]$. Then, by applying the same arguments step by step on $[t_k, t_{k+1})$, we conclude that there exists a strong solution for all $t \geq 0$. Similarly, for the case of the Neumann boundary conditions (3), the proof of well-posedness can be established by applying the same procedure. We can conclude that a strong solution exists for all $t \geq 0$ in the sense that

$$\begin{aligned} &z \in C([0,\infty); [H^1(\Omega)]^M) \cap \mathcal{L}^2([0,\infty); D(\tilde{\mathcal{A}})) \\ &z_t \in \mathcal{L}^2([0,\infty); [\mathcal{L}^2(\Omega)]^M) \end{aligned}$$

where

$$D(\tilde{\mathcal{A}}) = \left\{ g \in [H^2(\Omega)]^M \;\middle|\; \frac{\partial g}{\partial n}\Big|_{\partial\Omega} = 0 \right\}$$

5 | Stability Analysis of the Controlled System

Denote $f_j(x,t) \triangleq z(x,t) - \frac{\int_{\Omega_j} z(x,t)dx}{|\Omega_j|}$, $e_j(t) \triangleq \frac{\int_{\Omega_j}\int_{t_k}^t z_s(x,s)dsdx}{|\Omega_j|}$, $j = 1,2,\ldots,N_s$. We consider using the elementary relation

$$u_{\sigma_k}(t) = \begin{cases} 0, & t \in [t_k, t_k+\delta) \\ -K[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)], & t \in [t_k+\delta, t_{k+1}) \end{cases} \tag{15}$$

Under the controller (15) satisfying the switching law (12), the closed-loop system can be presented as two switches between three systems. The first one (for $t \in [t_k, t_k + \delta)$) is governed by

$$\begin{cases} z_t(x,t) = \Delta z(x,t) - \beta\nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t)) \\ \qquad -(1 - \chi_{[t_k,t_k+\delta]})b_{\sigma_k}(x)BK\left[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)\right], \\ \qquad x \in \Omega, t \in [t_k, t_{k+1}) \\ z(x,0) = z_0(x) \end{cases} \tag{16}$$

subject to (2) or (3). The second one (for $t \in [t_k + \delta, t_k + h_0)$) is governed by (16) where the ETM is not activated. The third one (for $t \in [t_k + h_0, t_{k+1})$) is governed by (16) subject to the continuous ETM (7).

Now we focus on the stability analysis of the above closed-loop system. To achieve this purpose, we construct the following Lyapunov-Krasovskii functional:

$$V(t) = V_{P_1}(t) + V_{P_3}(t) + \chi_{[t_k+\delta,\ t_k+h_0]}(t)V_R(t) + m(t), t \in [t_k, t_{k+1})$$

where

$$\begin{aligned}
V_{P_1}(t) &= \int_\Omega z^T(x,t)P_1 z(x,t)dx \\
V_{P_3}(t) &= \sum_{m=1}^{M} \int_\Omega (\nabla z_x^m(x,t))^T P_3 \nabla z_x^m(x,t)dx \\
V_R(t) &= (h_0 + t_k - t)\int_\Omega \int_{t_k+\delta}^{t} e^{-2\alpha(t-s)} z_s^T(x,s)Rz_s(x,s)dsdx
\end{aligned} \tag{17}$$

with $P_1 > 0$, $R > 0$, and positive diagonal $M \times M$-matrix $P_3 = \text{diag}\{p_3^1, \ldots, p_3^M\} > 0$. Note that $V_R(t_k+\delta) = V_R(t_k+h_0) = 0$. Moreover, due to (14), $V(t)$ is absolutely continuous in t.

The following results provide sufficient conditions in the form of the LMIs for the closed-loop system (16) under the Dirichlet boundary conditions:

Theorem 1. *Consider the closed-loop system (16) subject to Dirichlet boundary condition (2). Given positive parameters* $K,h_0,\theta,\varepsilon_0,\varepsilon,\alpha,\delta,\varepsilon_1 > \alpha, h_0 > \delta$ *and tuning parameter* α_0 *such that* $\alpha h_0 > (\alpha_0 + \alpha)\delta$, *let there exist* $M \times M$ *matrices* $P_1 > 0, P_2 > 0, P_3 = \text{diag}\{p_3^1, \ldots, p_3^M\} > 0, P_4 > 0, R > 0$, *and scalars* $\lambda_Q \geq 0, \lambda_i \geq 0$ $(i = 0,1,2)$ *that satisfy the following inequalities*:

$$2\alpha + \left(\frac{L}{N_s}\right)^N \lambda_2\theta - \varepsilon_0 < 0 \tag{18}$$

$$\Psi_m < 0, \quad m = 0,1,2,3 \tag{19}$$

where

$$\Psi_0 = \begin{bmatrix} -2\alpha_0 P_1 + \lambda_Q Q & P_1 + A^T P_3 & 0 & 0 \\ * & -2P_3 & -P_3\beta & P_3 \\ * & * & -2\alpha_0 P_3 \otimes I_N & 0 \\ * & * & * & -\lambda_Q I_M \end{bmatrix} \tag{20}$$

$$\Psi_1 = \begin{bmatrix} \Psi_{11}^1 & P_1 - \frac{P_2}{N_s-1} + A^T P_3 & -\frac{P_2\beta}{N_s-1} & \frac{P_2}{N_s-1} & \frac{P_2BK+K^TB^TP_2}{N_s-1} & \frac{P_2BK+K^TB^TP_2}{N_s-1} \\ * & (h_0-\delta)R - 2P_3 & -P_3\beta & P_3 & 0 & 0 \\ * & * & \Psi_{33}^1 & 0 & 0 & 0 \\ * & * & * & -\lambda_Q I_M & 0 & 0 \\ * & * & * & * & -\lambda_1 I_M - \frac{P_2BK+K^TB^TP_2}{N_s-1} & -\frac{P_2BK+K^TB^TP_2}{N_s-1} \\ * & * & * & * & * & -\frac{e^{-2\alpha(h_0-\delta)}}{h_0-\delta}R - \frac{P_2BK+K^TB^TP_2}{N_s-1} \end{bmatrix} \tag{21}$$

$$\Psi_2 = \begin{bmatrix} \Psi_{11}^2 & P_1 - P_4 + (A-BK)^T P_3 & -P_4\beta & P_4 & P_4BK - P_2BK - K^TB^TP_2 & P_4BK - P_2BK - K^TB^TP_2 \\ * & (h_0-\delta)R - 2P_3 & -P_3\beta & P_3 & P_3BK & P_3BK \\ * & * & \Psi_{33}^2 & 0 & 0 & 0 \\ * & * & * & -\lambda_Q I_M & 0 & 0 \\ * & * & * & * & -\lambda_1 I_M + P_2BK + K^TB^TP_2 & P_2BK + K^TB^TP_2 \\ * & * & * & * & * & -\frac{e^{-2\alpha(h_0-\delta)}}{h_0-\delta}R + P_2BK + K^TB^TP_2 \end{bmatrix} \tag{22}$$

$$\Psi_3 = \begin{bmatrix} \Psi_{11}^3 & P_1 - \frac{P_2}{N_s-1} + A^T P_3 & -\frac{P_2\beta}{N_s-1} & \frac{P_2}{N_s-1} & \frac{P_2BK+K^TB^TP_2}{N_s-1} & \frac{P_2BK+K^TB^TP_2}{N_s-1} \\ * & -2P_3 & -P_3\beta & P_3 & 0 & 0 \\ * & * & \Psi_{33}^3 & 0 & 0 & 0 \\ * & * & * & -\lambda_Q I_M & 0 & 0 \\ * & * & * & * & -\lambda_1 I_M - \frac{P_2BK+K^TB^TP_2}{N_s-1} & -\frac{P_2BK+K^TB^TP_2}{N_s-1} \\ * & * & * & * & * & \Psi_{66}^3 \end{bmatrix} \tag{23}$$

$$
\Psi_4 = \begin{bmatrix} \Psi_{11}^4 & P_1 - P_4 + (A-BK)^T P_3 & -P_4\beta & P_4 & P_4BK - P_2BK - K^TB^TP_2 & P_4BK - P_2BK - K^TB^TP_2 \\ * & -2P_3 & -P_3\beta & P_3 & P_3BK & P_3BK \\ * & * & \Psi_{33}^4 & 0 & 0 & 0 \\ * & * & * & -\lambda_Q I_M & 0 & 0 \\ * & * & * & * & -\lambda_1 I_M + P_2BK + K^TB^TP_2 & P_2BK + K^TB^TP_2 \\ * & * & * & * & * & \Psi_{66}^4 \end{bmatrix} \tag{24}
$$

$$
\Psi_{11}^1 = \frac{1}{N_s - 1}[P_2(A-BK) + (A-BK)^T P_2] + 2\alpha P_1 + \lambda_Q Q - \lambda_0 I_M
$$

$$
\Psi_{33}^1 = \Psi_{33}^3 = \left(\lambda_1 \frac{NL^2}{N_s^2\pi^2} I_M + \lambda_0 \frac{L^2}{N\pi^2} I_M + 2\alpha P_3 - \frac{2}{N_s - 1} P_2\right) \otimes I_N
$$

$$
\Psi_{11}^2 = P_4(A-BK) + (A-BK)^T P_4 + P_2BK + K^TB^TP_2 + 2\alpha P_1 + \lambda_Q Q - \lambda_0 I_M
$$

$$
\Psi_{33}^2 = \Psi_{33}^4 = \left(\lambda_1 \frac{NL^2}{N_s^2\pi^2} I_M + \lambda_0 \frac{L^2}{N\pi^2} I_M + 2\alpha P_3 - 2P_4\right) \otimes I_N
$$

$$
\Psi_{11}^3 = \frac{1}{N_s - 1}[P_2(A-BK) + (A-BK)^T P_2] + 2\alpha P_1 + \lambda_Q Q - \lambda_0 I_M + \lambda_2 \varepsilon I_M + \varepsilon \left(\frac{N_s}{L}\right)^N I_M
$$

$$
\Psi_{66}^3 = -\lambda_2 I_M - \left(\frac{N_s}{L}\right)^N I_M - \frac{P_2BK + K^TB^TP_2}{N_s - 1}
$$

$$
\Psi_{11}^4 = P_4(A-BK) + (A-BK)^T P_4 + P_2BK + K^TB^TP_2 + 2\alpha P_1 + \lambda_Q Q - \lambda_0 I_M + \lambda_2 \varepsilon I_M + \varepsilon \left(\frac{N_s}{L}\right)^N I_M
$$

$$
\Psi_{66}^4 = -\lambda_2 I_M - \left(\frac{N_s}{L}\right)^N I_M + P_2BK + K^TB^TP_2
$$

Let α_1 be subject to

$$
0 < \alpha_1 h_0 < \alpha h_0 - (\alpha_0 + \alpha)\delta \tag{25}
$$

Then the closed-loop system (*16*) *subject to* (*2*) *is exponentially stable.*

Proof. **Step 1:** On the moving time interval $[t_k, t_k + \delta)$, we have the open-loop system. We first derive sufficient LMI-based conditions to guarantee that $\dot{V}(t) - 2\alpha_0 V(t) \le 0$ for $[t_k, t_k + \delta)$.

Taking the time derivative of $V(t)$ along the trajectory of the open-loop system (16) subject to (2), for $t \in [t_k, t_k + \delta)$ we have

$$
\begin{aligned}
\dot{V}(t) - 2\alpha_0 V(t) &= \dot{V}_{P_1}(t) + \dot{V}_{P_3}(t) + \dot{m}(t) - 2\alpha_0[V_{P_1}(t) + V_{P_3}(t) + m(t)] \\
&= 2\int_\Omega z^T(x,t) P_1 z_t(x,t) dx + 2\sum_{m=1}^{M} \int_\Omega p_3^m z_x^m(x,t) z_{xt}^m(x,t) dx \\
&\quad - 2\alpha_0 \int_\Omega z^T(x,t) P_1 z(x,t) dx - 2\alpha_0 \sum_{m=1}^{M} \int_\Omega (\nabla z_x^m(x,t))^T P_3 \nabla z_x^m(x,t) dx \\
&\quad - 2(\varepsilon_1 + \alpha_0) m(t)
\end{aligned}
$$

From (4), we have

$$
\lambda_Q \int_\Omega [z^T(x,t) Q z(x,t) - \varphi^T(z(x,t)) \varphi(z(x,t))] dx \ge 0 \tag{26}
$$

for any $\lambda_Q \ge 0$.

We apply the descriptor method of Reference [34] to (16), where the left-hand side of the following equation

$$
2\int_\Omega z_t^T(x,t) P_3 [-z_t(x,t) + \Delta z(x,t) - \beta \nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t))] dx = 0 \tag{27}
$$

is added to $\dot{V}(t) - 2\alpha_0 V(t)$.

Set $\eta_0 = \text{col}\{z(x,t), z_t(x,t), \nabla_x z(x,t), \varphi(z(x,t))\}$. Therefore, we have

$$\begin{aligned}\dot V(t) - 2\alpha_0 V(t) \le& \int_\Omega z^T(x,t)\left[-2\alpha_0 P_1 + \lambda_Q Q\right] z(x,t)dx \\ &- 2\int_\Omega z_t^T(x,t)P_3 z_t(x,t)dx + \int_\Omega \nabla_x^T z(x,t)\left[-2\alpha_0 P_3 \otimes I_N\right]\nabla_x z(x,t)dx \\ &- \lambda_Q \int_\Omega \varphi^T(z(x,t))\varphi(z(x,t))dx + \int_\Omega z^T(x,t)\left[P_1 + A^T P_3\right] z_t(x,t)dx \\ &+ 2\int_\Omega z_t^T(x,t)P_3\left[-\beta\nabla_x z(x,t) + \varphi(z(x,t))\right]dx \\ \le& \int_\Omega \eta_0^T \Psi_0 \eta_0 dx \le 0, t \in [t_k, t_k+\delta)\end{aligned} \tag{28}$$

if $\Psi_0 \le 0$ holds.

Step 2: On the waiting time interval $[t_k+\delta, t_k+h_0)$, the ETM is not activated. To obtain the maximal value of the waiting time, we derive sufficient conditions to guarantee that $\dot V(t) + 2\alpha V(t) \le 0$.

Differentiating along the closed-loop system (16) subject to (2) and (12), we have

$$\begin{aligned}\dot V(t) + 2\alpha V(t) =& 2\int_\Omega z^T(x,t)P_1 z_t(x,t)dx + 2\sum_{m=1}^M \int_\Omega p_3^m z_x^m(x,t) z_{xt}^m(x,t)dx \\ &+ (h_0 + t_k - t)\int_\Omega z_t^T(x,t) R z_t(x,t)dx \\ &- \int_\Omega \int_{t_k+\delta}^t e^{-2\alpha(t-s)} z_s^T(x,s) R z_s(x,s)dsdx \\ &+ 2\alpha\int_\Omega z^T(x,t)P_1 z(x,t)dx + 2\alpha\sum_{m=1}^M \int_\Omega (\nabla z_x^m(x,t))^T P_3 \nabla z_x^m(x,t)dx \\ &+ 2(\alpha - \epsilon_1)m(t)\end{aligned} \tag{29}$$

Jensen's inequality leads to

$$\begin{aligned}&- \int_\Omega \int_{t_k+\delta}^t e^{-2\alpha(t-s)} z_s^T(x,s) R z_s(x,s)dsdx \\ &\le \frac{-e^{-2\alpha(h_0-\delta)}}{h_0-\delta}\int_\Omega [z(x,t) - z(x,t_k)]^T R[z(x,t) - z(x,t_k)]dx \\ &\le \frac{-e^{-2\alpha(h_0-\delta)}}{h_0-\delta}\sum_{j=1}^{N_s}\int_{\Omega_j} e_j^T(t) R e_j(t)dx\end{aligned} \tag{30}$$

Wirtinger's inequality yields

$$\lambda_0\left[\frac{L^2}{N\pi^2}\|\nabla_x z\|^2_{\mathcal{L}^2(\Omega)} - \|z\|^2_{\mathcal{L}^2(\Omega)}\right] \ge 0 \tag{31}$$

for any $\lambda_0 \ge 0$.

Since $\int_{\Omega_j} f_j(x,t)dx = 0$, Poincaré's inequality leads to

$$\lambda_1 \sum_{j=1}^{N_s}\left[\frac{NL^2}{N_s^2\pi^2}\int_{\Omega_j}|\nabla_x z(x,t)|^2 dx - \int_{\Omega_j}|f_j(x,t)|^2 dx\right] \ge 0 \tag{32}$$

where $\lambda_1 \ge 0$.

Note that $|\Omega_j| = \left(\frac{L}{N_s}\right)^N$, $j = 1, \dots, N_s$. Then integrating (12) over the subdomain Ω_j, we have

$$\begin{aligned}&\frac{1}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}[z(x,t) - f_j(x,t) - e_j(t)]^T \\ &\quad\times(P_2BK + K^T B^T P_2)[z(x,t) - f_j(x,t) - e_j(t)]dx \\ &\le \int_{\Omega_{\sigma_k}}[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)]^T \\ &\quad\times(P_2BK + K^T B^T P_2)[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)]dx\end{aligned} \tag{33}$$

We apply the descriptor method of Reference [34] to (33), where the left-hand side of the following equations

$$\begin{aligned}&\frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)P_2[-z_t(x,t) + \Delta z(x,t) - \beta\nabla_x z(x,t) \\ &\quad + Az(x,t) + \varphi(z(x,t))]dx = 0 \\ &2\int_{\Omega_{\sigma_k}} z^T(x,t)P_4\big\{-z_t(x,t) + \Delta z(x,t) - \beta\nabla_x z(x,t) \\ &\quad + Az(x,t) + \varphi(z(x,t)) - BK[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)]\big\}dx = 0\end{aligned}$$

with $P_2 > 0$ and $P_4 > 0$ are added to (33). Then one has

$$\begin{aligned}&\frac{1}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)[P_2(A-BK) + (A-BK)^T P_2]z(x,t)dx \\ &- \frac{1}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} f_j^T(x,t)(P_2BK + K^T B^T P_2)f_j(x,t)dx \\ &- \frac{1}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} e_j^T(t)(P_2BK + K^T B^T P_2)e_j(t)dx \\ &+ \frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)(P_2BK + K^T B^T P_2)[f_j(x,t) + e_j(t)]dx \\ &- \frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} f_j^T(x,t)(P_2BK + K^T B^T P_2)e_j(t)dx \\ &+ \frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)P_2[-z_t(x,t) + \Delta z(x,t) - \beta\nabla_x z(x,t) + \varphi(z(x,t))]dx \\ &+ \int_{\Omega_{\sigma_k}} z^T(x,t)[P_4(A-BK) + (A-BK)^T P_4 + P_2BK + K^T B^T P_2]z(x,t)dx \\ &+ \int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(P_2BK + K^T B^T P_2)f_{\sigma_k}(x,t) \\ &+ \int_{\Omega_{\sigma_k}} e_{\sigma_k}^T(t)(P_2BK + K^T B^T P_2)e_{\sigma_k}(t) \\ &+ 2\int_{\Omega_{\sigma_k}} z^T(x,t)(P_4BK - P_2BK - K^T B^T P_2)[f_{\sigma_k}(x,t) + e_{\sigma_k}(t)]dx \\ &+ 2\int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(P_2BK + K^T B^T P_2)e_{\sigma_k}(t)dx \\ &+ 2\int_{\Omega_{\sigma_k}} z^T(x,t)P_4[-z_t(x,t) + \Delta z(x,t) - \beta\nabla_x z(x,t) + \varphi(z(x,t))]dx \ge 0\end{aligned} \tag{34}$$

We apply further the descriptor method to (16), where the left-hand side of the following equation

$$\begin{aligned}&2\int_\Omega z_t^T(x,t)P_3\big\{-z_t(x,t) + \Delta z(x,t) - \beta\nabla_x z(x,t) \\ &+ Az(x,t) + \varphi(z(x,t)) - b_{\sigma_k}(x)BK[z(x,t) - f_{\sigma_k}(x,t) - e_{\sigma_k}(t)]\big\}dx = 0\end{aligned} \tag{35}$$

is added to $\dot V(t) + 2\alpha V(t)$.

From (26) and (29–33), we have

$$
\begin{aligned}
&\dot V(t)+2\alpha V(t)\\
&\le \sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)\Big\{\frac{1}{N_s-1}[P_2(A-BK)+(A-BK)^TP_2]\\
&+2\alpha P_1+\lambda_Q Q-\lambda_0 I_M\Big\}z(x,t)dx+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z_t^T(x,t)[(h_0-\delta)R-2P_3]z_t(x,t)dx\\
&+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} e_j^T(t)\left[-\frac{e^{-2\alpha(h_0-\delta)}}{h_0-\delta}R-\frac{P_2BK+K^TB^TP_2}{N_s-1}\right]e_j(t)dx\\
&+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} f_j^T(x,t)\left[-\lambda_1 I_M-\frac{P_2BK+K^TB^TP_2}{N_s-1}\right]f_j(x,t)dx\\
&-\lambda_Q\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\varphi^T(z(x,t))\varphi(z(x,t))dx+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\nabla_x^T z(x,t)\\
&\left[\lambda_1\frac{NL^2}{N_s^2\pi^2}I_M+\lambda_0\frac{L^2}{N\pi^2}I_M+2\alpha P_3-\frac{2}{N_s-1}P_2\right]\otimes I_N\nabla_x z(x,t)dx\\
&+2\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)\left[P_1-\frac{1}{N_s-1}P_2+A^TP_3\right]z_t(x,t)dx\\
&+\frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} z^T(x,t)(P_2BK+K^TB^TP_2)[f_j(x,t)+e_j(t)]dx\\
&-\frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j} f_j^T(x,t)(P_2BK+K^TB^TP_2)e_j(t)dx\\
&+2\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\left[\frac{z^T(x,t)P_2}{N_s-1}+z_t^T(x,t)P_3\right][-\beta\nabla_x z(x,t)+\varphi(z(x,t))]dx\\
&+\int_{\Omega_{\sigma_k}} z^T(x,t)[P_4(A-BK)+(A-BK)^TP_4\\
&+P_2BK+K^TB^TP_2+2\alpha P_1+\lambda_Q Q-\lambda_0 I_M]z(x,t)dx\\
&+\int_{\Omega_{\sigma_k}} z_t^T(x,t)[(h_0-\delta)R-2P_3]z_t(x,t)dx\\
&+\int_{\Omega_{\sigma_k}} e_{\sigma_k}^T(t)\left[-\frac{e^{-2\alpha(h_0-\delta)}}{h_0-\delta}R+P_2BK+K^TB^TP_2\right]e_{\sigma_k}(t)dx\\
&+\int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(-\lambda_1 I_M+P_2BK+K^TB^TP_2)f_{\sigma_k}(x,t)dx\\
&-\lambda_Q\int_{\Omega_{\sigma_k}}\varphi^T(z(x,t))\varphi(z(x,t))dx\\
&+\int_{\Omega_{\sigma_k}}\nabla_x^T z(x,t)\left[\lambda_1\frac{NL^2}{N_s^2\pi^2}I_M+\lambda_0\frac{L^2}{N\pi^2}I_M+2\alpha P_3-2P_4\right]\otimes I_N\nabla_x z(x,t)dx\\
&+2\int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(P_2BK+K^TB^TP_2)e_{\sigma_k}(t)dx\\
&+2\int_{\Omega_{\sigma_k}} z^T(x,t)[P_1-P_4+(A-BK)^TP_3]z_t(x,t)dx\\
&+2\int_{\Omega_{\sigma_k}} z^T(x,t)(P_4BK-P_2BK-K^TB^TP_2)[f_{\sigma_k}(x,t)+e_{\sigma_k}(t)]dx\\
&+2\int_{\Omega_{\sigma_k}}\left[z^T(x,t)P_4+z_t^T(x,t)P_3\right]\{-\beta\nabla_x z(x,t)+\varphi(z(x,t))\}dx\\
&+2\int_{\Omega_{\sigma_k}} z_t^T(x,t)P_3BK[f_{\sigma_k}(x,t)+e_{\sigma_k}(t)]dx+2(\alpha-\varepsilon_1)m(t),\ t\in[t_k+\delta,t_k+h_0)
\end{aligned}
\tag{36}
$$

Set $\eta_1=\mathrm{col}\{z(x,t),z_t(x,t),\nabla_x z(x,t),\varphi(z(x,t)),f_j(x,t),e_j(t)\}$, $j\neq\sigma_k$ and $\eta_2=\mathrm{col}\{z(x,t),z_t(x,t),\nabla_x z(x,t),\varphi(z(x,t)),f_{\sigma_k}(x,t),e_{\sigma_k}(t)\}$. Then we have

$$
\dot V(t)+2\alpha V(t)\le\sum_{j\neq\sigma_k}^{N}\int_{\Omega_j}\eta_1^T\Psi_1\eta_1dx+\int_{\Omega_{\sigma_k}}\eta_2^T\Psi_2\eta_2dx+2(\alpha-\varepsilon_1)m(t),\ t\in[t_k+\delta,t_k+h_0)
$$

where Ψ_1 and Ψ_2 are given by (21) and (22).

Step 3: On the remaining part of the time interval $[t_k+h_0,t_{k+1})$, the ETM is activated. We derive sufficient LMI-based conditions to guarantee that $\dot V(t)+2\alpha V(t)\le 0$.

For (6), Jensen's inequality leads to

$$
|y_j(t)|^2=\left[\frac{\int_{\Omega_j}z(x,t)dx}{|\Omega_j|}\right]^2\le\frac{1}{|\Omega_j|}\int_{\Omega_j}z^2(x,t)dx
$$

Then above inequality and the event-triggering condition (7) yield

$$
\begin{aligned}
0&<\sum_{j=1}^{N_s}\left[\varepsilon\left|y_j(t)\right|^2-\left|e_j(t)\right|^2\right]+\theta m(t)\\
&\le\sum_{j=1}^{N_s}\frac{\varepsilon}{|\Omega_j|}\int_{\Omega_j}z^2(x,t)dx-\sum_{j=1}^{N_s}\frac{1}{|\Omega_j|}\int_{\Omega_j}\left|e_j(t)\right|^2dx+\theta m(t)
\end{aligned}
$$

Thus, for $\lambda_2\ge 0$, we have

$$
\lambda_2\sum_{j=1}^{N_s}\left\{\int_{\Omega_j}\left[\varepsilon|z(x,t)|^2-|e_j(t)|^2\right]dx\right\}+\lambda_2\left(\frac{L}{N_s}\right)^N\theta m(t)>0
\tag{37}
$$

Differentiating along the closed-loop system (16) subject to (2), (7), and (12), we get

$$
\begin{aligned}
&\dot V(t)+2\alpha V(t)\\
&\le\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}z^T(x,t)\Big\{\frac{P_2(A-BK)+(A-BK)^TP_2}{N_s-1}+2\alpha P_1+\lambda_Q Q-\lambda_0 I_M\\
&+\lambda_2\varepsilon I_M+\varepsilon\left(\frac{N_s}{L}\right)^N I_M\Big\}z(x,t)dx-2\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}z_t^T(x,t)P_3z_t(x,t)dx\\
&+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}e_j^T(t)\left[-\lambda_2 I_M-\left(\frac{N_s}{L}\right)^N I_M-\frac{P_2BK+K^TB^TP_2}{N_s-1}\right]e_j(t)dx\\
&+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}f_j^T(x,t)\left[-\lambda_1 I_M-\frac{1}{N_s-1}(P_2BK+K^TB^TP_2)\right]f_j(x,t)dx\\
&-\lambda_Q\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\varphi^T(z(x,t))\varphi(z(x,t))dx+\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\nabla_x^T z(x,t)\\
&\times\left[\lambda_1\frac{NL^2}{N_s^2\pi^2}I_M+\lambda_0\frac{L^2}{N\pi^2}I_M+2\alpha P_3-\frac{2}{N_s-1}P_2\right]\otimes I_N\nabla_x z(x,t)dx\\
&+2\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}z^T(x,t)\left[P_1-\frac{1}{N_s-1}P_2+A^TP_3\right]z_t(x,t)dx\\
&+\frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}z^T(x,t)(P_2BK+K^TB^TP_2)[f_j(x,t)+e_j(t)]dx\\
&-\frac{2}{N_s-1}\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}f_j^T(x,t)(P_2BK+K^TB^TP_2)e_j(t)dx\\
&+2\sum_{j\neq\sigma_k}^{N_s}\int_{\Omega_j}\left[\frac{z^T(x,t)P_2}{N_s-1}+z_t^T(x,t)P_3\right]\{-\beta\nabla_x z(x,t)+\varphi(z(x,t))\}dx\\
&+\int_{\Omega_{\sigma_k}}z^T(x,t)[P_4(A-BK)+(A-BK)^TP_4+P_2BK+K^TB^TP_2+2\alpha P_1\\
&+\lambda_Q Q-\lambda_0 I_M+\lambda_2\varepsilon I_M+\varepsilon\left(\frac{N_s}{L}\right)^N I_M\Big]z(x,t)dx-2\int_{\Omega_{\sigma_k}}z_t^T(x,t)P_3z_t(x,t)dx\\
&+\int_{\Omega_{\sigma_k}}e_{\sigma_k}^T(t)\left[-\lambda_2 I_M-\left(\frac{N_s}{L}\right)^N I_M+P_2BK+K^TB^TP_2\right]e_{\sigma_k}(t)dx
\end{aligned}
$$

$$+ \int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(-\lambda_1 I_M + P_2 BK + K^T B^T P_2) f_{\sigma_k}(x,t) dx$$

$$- \lambda_Q \int_{\Omega_{\sigma_k}} \varphi^T(z(x,t))\varphi(z(x,t)) dx + \left[2\alpha + \left(\frac{L}{N_s}\right)^N \lambda_2\theta - \varepsilon_0\right] m(t)$$

$$+ \int_{\Omega_{\sigma_k}} \nabla_x^T z(x,t)\left[\lambda_1 \frac{NL^2}{N_s^2\pi^2} I_M + \lambda_0 \frac{L^2}{N\pi^2} I_M + 2\alpha P_3 - 2P_4\right] \otimes I_N \nabla_x z(x,t) dx$$

$$+ 2\int_{\Omega_{\sigma_k}} f_{\sigma_k}^T(x,t)(P_2 BK + K^T B^T P_2) e_{\sigma_k}(t) dx$$

$$+ 2\int_{\Omega_{\sigma_k}} z^T(x,t)[P_1 - P_4 + (A-BK)^T P_3] z_t(x,t) dx$$

$$+ 2\int_{\Omega_{\sigma_k}} z^T(x,t)(P_4 BK - P_2 BK - K^T B^T P_2)[f_{\sigma_k}(x,t) + e_{\sigma_k}(t)] dx$$

$$+ 2\int_{\Omega_{\sigma_k}} \left[z^T(x,t)P_4 + z_t^T(x,t)P_3\right]\left\{-\beta\nabla_x z(x,t) + \varphi(z(x,t))\right\} dx$$

$$+ 2\int_{\Omega_{\sigma_k}} z_t^T(x,t) P_3 BK[f_{\sigma_k}(x,t) + e_{\sigma_k}(t)] dx, t \in [t_k + h_0, t_{k+1})$$

Then we have

$$\dot V(t) + 2\alpha V(t) \le \sum_{j \ne \sigma_k}^{N} \int_{\Omega_j} \eta_1^T \Psi_3 \eta_1 dx + \int_{\Omega_{\sigma_k}} \eta_2^T \Psi_4 \eta_2 dx + \left[2\alpha + \left(\frac{L}{N_s}\right)^N \lambda_2\theta - \varepsilon_0\right] m(t), t \in [t_k + h_0, t_{k+1})$$

where Ψ_3 and Ψ_4 are given by (23) and (24).

Step 4: From Step 1-Step 3, we obtain the feasibility of LMIs (18) and (19) that implies that any strong solution of (16), (2) initialized with z_0 admits a priori estimate

- $V(t) \le e^{2\alpha_0(t-t_k)} V(t_k), \ \forall t \in [t_k, t_k + \delta)$,
- $V(t) \le e^{-2\alpha(t-t_k-\delta)} V(t_k + \delta), \ \forall t \in [t_k + \delta, t_{k+1})$.

Since $\alpha_1 < \alpha$ and $t_{k+1} - t_k \ge h_0$, (25) implies

$$(\alpha_1 - \alpha)(t_{k+1} - t_k) \le (\alpha_1 - \alpha)h_0 \le -(\alpha_0 + \alpha)\delta \tag{38}$$

Hence,

$$V(t_{k+1}) \le e^{2\alpha_0\delta - 2\alpha(t_{k+1}-t_k-\delta)} V(t_k) \le e^{-2\alpha_1(t_{k+1}-t_k)} V(t_k)$$

if $0 < \alpha_1 h_0 \le \alpha h_0 - (\alpha_0 + \alpha)\delta$.

For $t \in [t_k, t_k + \delta)$,

$$V(t) \le e^{2\alpha_0\delta} V(t_k)$$

For $t \in [t_k + \delta, t_{k+1})$,

$$V(t) \le V(t_k + \delta) \le e^{2\alpha_0\delta} V(t_k)$$

Therefore, we have $V(t) \le e^{2\alpha_0\delta} V(t_k) \le e^{2\alpha_0\delta - 2\alpha_1(t_k - t_{k-1})} V(t_{k-1}) \le \cdots \le e^{2\alpha_0\delta - 2\alpha_1 t_k} V(0), \ t \in [t_k, t_{k+1})$.

Then using the step method of Reference [34], we conclude that the strong solution exists for all $t \ge 0$. Moreover, the closed-loop system is exponentially stable. □

Under the Neumann boundary conditions, the result is similar:

Theorem 2. *Consider the closed-loop system (16) subject to Neumann boundary condition (3). Given positive parameters $K, h_0, \theta, \varepsilon_0, \varepsilon, \alpha, \delta, \varepsilon_1 > \alpha, h_0 > \delta$, and tuning parameter α_0 such that $\alpha h_0 > (\alpha_0 + \alpha)\delta$, let there exist $M \times M$ matrices $P_1 > 0, P_2 > 0, P_3 = \text{diag}\{p_3^1, \dots, p_3^M\} > 0, P_4 > 0, R > 0$, and scalars $\lambda_Q \ge 0, \lambda_i \ge 0$ $(i = 1, 2)$ that satisfy (18)* and *(19) with $\lambda_0 = 0$. Let α_1 be subject to (25). Then the closed-loop system (16) subject to (3) is exponentially stable.*

Remark 2. The proof of Theorem 2 is omitted. For the case of Neumann boundary conditions, Wirtinger's inequality (Lemma 1) is not applicable. Hence, if the LMI conditions of Theorem 1 with $\lambda_0 = 0$ hold, then the closed-loop system (16) under the Neumann boundary conditions (3) is exponentially stable.

Remark 3. As for the feasibility of the LMI conditions of Theorem 1, we first show that strict inequalities $\Psi_m < 0$ with $m = 0, 1, 2, 3, 4$ hold with $\delta = h_0 = \varepsilon = \beta = \alpha = Q = \lambda_0 = 0$, and large enough λ_Q, λ_1, N_s, α_0. Then LMIs $\Psi_m < 0$ with $m = 0, 1, 2, 3, 4$ hold with small enough $\delta > 0$, $h_0 > 0$, $\varepsilon > 0$, $\|\beta\|_2 > 0$, $\alpha > 0$, $Q > 0$, $\lambda_0 > 0$, and large enough λ_Q, λ_1, N_s, α_0. Since (A, B) is controllable, there exists K such that $A - BK$ is Hurwitz. Let $P_1 > 0$ be the solution to the equation $P_1(A - BK) + (A - BK)^T P_1 = -\gamma I_M$ for some large enough scalar $\gamma > 0$. Set $R = I_M$, $P_2 = \frac{1}{N_s} P_1$, and $P_3 = P_4 = P_1$. Then for large enough N_s, λ_Q, the Schur complement theorem with $\delta = h_0 = \varepsilon = \beta = \alpha = Q = \lambda_0 = 0$ leads to

$$\begin{aligned} &\Psi_0 < 0 \Leftrightarrow 2\alpha_0 P_1 - (P_1 + A^T P_1)(2P_1)^{-1}(P_1 + P_1 A) > 0 \\ &\Psi_1 < 0, \Psi_3 < 0 \Leftrightarrow -\Psi_{11}^1 - (P_1 + A^T P_1)(2P_1)^{-1}(P_1 + P_1 A) > 0, \Psi_{33}^1 < 0 \\ &\Leftrightarrow \frac{\gamma I_M}{N_s(N_s - 1)} - (P_1 + A^T P_1)(2P_1)^{-1}(P_1 + P_1 A) > 0, \Psi_{33}^1 < 0 \end{aligned} \tag{39}$$

$$\begin{aligned} &\Psi_2 < 0, \Psi_4 < 0 \Leftrightarrow -\Psi_{11}^2 - (A - BK)^T P_1(2P_1)^{-1} P_1(A - BK) > 0, \Psi_{33}^2 < 0 \\ &\Leftrightarrow \gamma I_M - \frac{1}{N_s}(P_1 BK + K^T B^T P_1) - (A - BK)^T P_1(2P_1)^{-1} P_1(A - BK) > 0, \Psi_{33}^2 < 0 \end{aligned} \tag{40}$$

Here we use the fact that $\Psi_{33}^1 = \Psi_{33}^3$, $\Psi_{33}^2 = \Psi_{33}^4$ and for $\varepsilon = 0$, $\Psi_{11}^1 = \Psi_{11}^3, \Psi_{11}^2 = \Psi_{11}^4$. Note that $\Psi_0 < 0$ holds with a large enough α_0. Given $\alpha_1 < \alpha$, the inequalities (18) and (25) hold with appropriate $\varepsilon_0 > 2\alpha$ for small enough δ and θ. Choose large enough γ, λ_1, and N_s such that $\lambda_1 \frac{NL^2}{N_s^2\pi^2}$ is small, $\frac{\gamma}{N_s(N_s-1)}$ is large, and (39), (40) hold. Therefore, the LMI conditions of Theorem 1 are always feasible for small enough δ, h_0, ε, $\|\beta\|_2$, α, Q, λ_0, θ and large enough λ_Q, λ_1, N_s, α_0, γ. It should be noticed that the above analysis with $\lambda_0 = 0$ guarantees the feasibility of the LMI conditions of Theorem 2.

6 | Simulation Example

Consider the semilinear 2-D reaction-convection-diffusion PDE with $M = 2$ and $m = 2$ as follows:

$$\begin{cases} z_t(x,t) = \Delta z(x,t) - \beta\nabla_x z(x,t) + Az(x,t) + \varphi(z(x,t)) \\ \qquad + b_{\sigma_k}(x) B u_{\sigma_k}(t), x \in \Omega = [0,1] \times [0,1] \\ z(x,0) = z_0(x) \end{cases} \tag{41}$$

subject to Dirichlet boundary condition (2), where $z_0(x) = [\sin(\pi x_1)\sin(\pi x_2), 2\sin(\pi x_1)\sin(\pi x_2)]^T$, $\varphi(z(x,t)) = [0.01(1 - e^{-[z^1(x,t)]^2}), 0]^T$ and

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 0.01 & 0.01 & 0.05 & 0 \\ 0 & 0.05 & 0.01 & 0.01 \end{bmatrix}, K = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$$

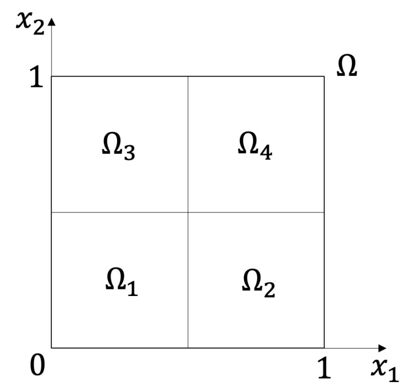

FIGURE 1 | Domain Ω and subdomains Ω_j.

Set $N_s = 4$. This means that Ω is divided into four equal subdomains (see Figure 1). As seen from Figure 2, the open-loop system (41) is unstable.

Some parameters of the dynamic ETM (7) are selected as $h_0 = 0.1$, $\epsilon = 0.0001$, $\epsilon_0 = 2.5$, $\epsilon_1 = 0.1$, $\theta = 0.0001$, and $m_0 = 10$. Choose $\alpha = 0.09$, $\alpha_0 = 4.8$, $\alpha_1 = 0.01$, and $\delta = 0.001$. By Yalmip, the feasible solutions of LMI conditions of Theorem 1 can be obtained as follows:

$$\lambda_Q = 240.7294, \lambda_0 = 33.1760, \lambda_1 = 67.9458, \lambda_2 = 1.1532 \times 10^3$$

$$P_1 = \begin{bmatrix} 5.5853 & -0.0806 \\ -0.0806 & 5.8717 \end{bmatrix}, P_2 = \begin{bmatrix} 3.8497 & -0.0136 \\ -0.0136 & 4.4565 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0.9628 & 0 \\ 0 & 1.6054 \end{bmatrix}, P_4 = \begin{bmatrix} 11.5544 & -0.0857 \\ -0.0857 & 10.9844 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.1028 & 0.0357 \\ 0.0357 & 0.3481 \end{bmatrix}$$

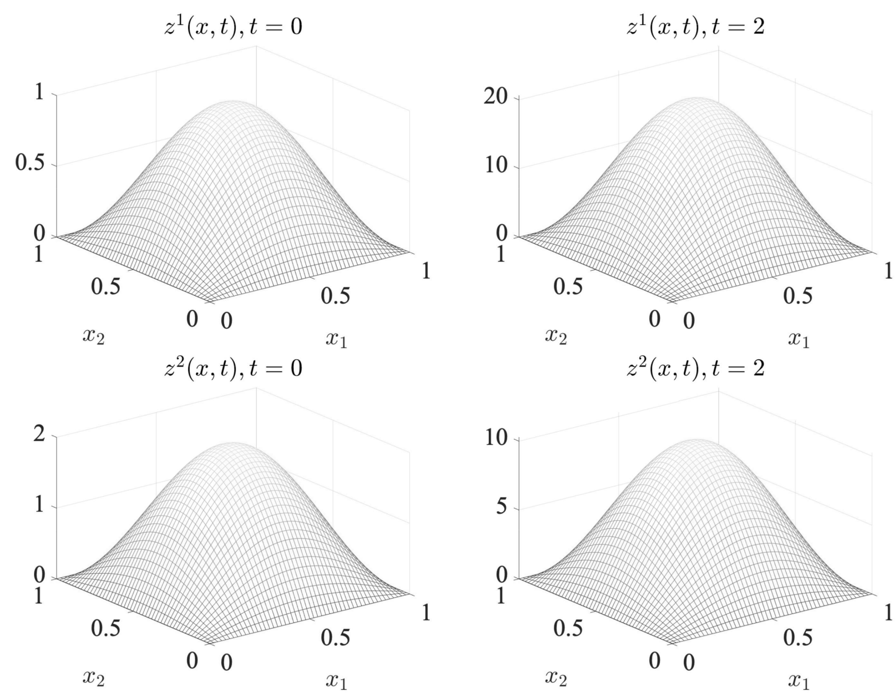

FIGURE 2 | State of the open-loop system (41).

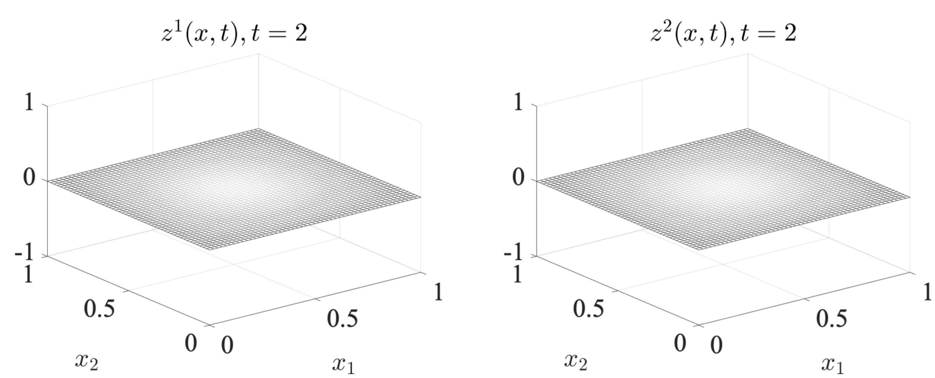

FIGURE 3 | State of the closed-loop system (41).

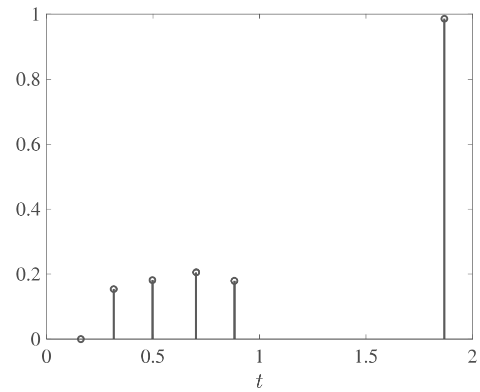

FIGURE 4 | Release time and interval of dynamic ETM (7).

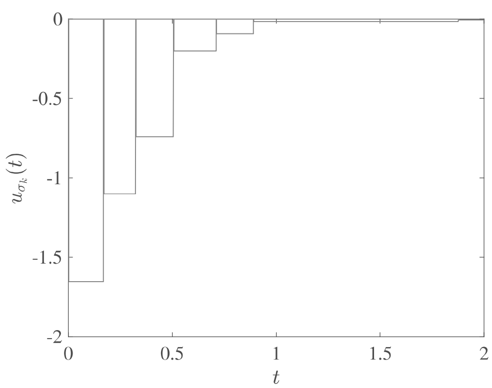

FIGURE 5 | Control input (9).

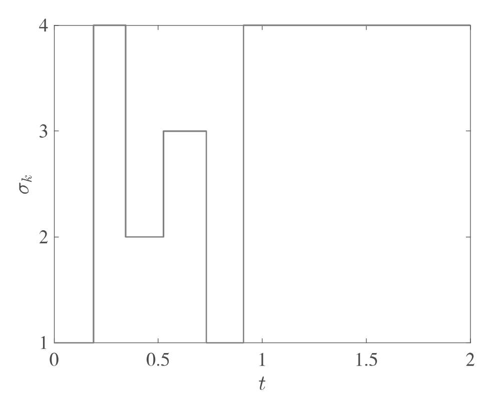

FIGURE 6 | Locations of the actuator.

The steps in time and space are chosen as $dx = 0.02$ and $dt = 0.0002$, respectively. The finite difference method is adopted to compute the solutions of the closed-loop system (41). Under the proposed dynamic event-triggered switching controller (9) subject to the switching rule (12) and dynamic ETM (7), the closed-loop system (41) is stable as Figure 3, which illustrates the effectiveness of our proposed methods. Figure 4 depicts the release time and interval of dynamic ETM (7). Figure 5 shows the trajectory of control input (9). The locations of the actuator are described in Figure 6.

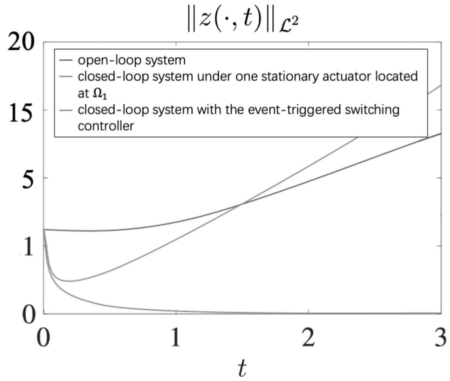

FIGURE 7 | $\|z(\cdot,t)\|_{\mathcal{L}^2}$ of the open-loop system, closed-loop system under one stationary actuator located at Ω_1, and closed-loop system with the dynamic event-triggered switching controller (9).

TABLE 1 | Comparison with different values of ε, θ, and m_0.

ε	θ	m_0	Sent measurements	Maximal value of h_0
0.001	0.0001	10	7	0.1003
0.0005	0.0001	10	8	0.1034
0.0001	0.0001	10	9	0.1142
0.0001	0.0005	10	8	0.1142
0.0001	0.001	10	7	0.1142
0.0001	0.001	25	6	0.1142
0.0001	0.001	75	5	0.1142
0	0	0	20	0.1504

Figure 7 shows the time evolution of $\|z(\cdot,t)\|_{\mathcal{L}^2}$ of the closed-loop and open-loop systems. The closed-loop system with the proposed dynamic event-triggered switching controller (9) is stable, whereas the closed-loop system under only one stationary actuator is unstable.

With different values of ε, θ, and m_0 in dynamic ETM (7), the amount of sent measurements and the maximal values of h_0 are shown in Table 1. The simulation results indicate that the sent measurements are reduced by choosing larger ε, θ, and m_0. The maximal value of h_0 is decreased with a larger ε. The last line in Table 1 (with $\varepsilon = \varepsilon_0 = \varepsilon_1 = \theta = 0$) corresponds to time-triggered switching control (considered in our previous work [20]). It is seen that the suggested ETM reduces the number of sent measurements (5 compared with 20) at least by 4 times.

7 | Conclusions

In this paper, stabilization of the N-D semilinear reaction-convection-diffusion equation under the event-triggered controller by using switching has been considered. Sufficient conditions have been provided by means of LMIs that ensure the corresponding closed-loop system is exponentially stable. In the future work, we may focus on the study of the event-triggered controller design for 2-D parabolic PDEs under pointlike measurements or stochastic PDE systems and coupled PDE-PDE systems by switching.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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